UNL Professional Math and Science Institute
Lesson Plan
Using Logger Pro to Analyze Crash Test Video

The lesson plan is designed to allow students to learn to use Logger Pro to analyze video with the end product being the analysis of a crash test video followed by a discussion of the effectiveness and designs of road safety barriers.

State Mathematics Standards:

12.4.5 By the end of twelfth grade, students will apply right triangle trigonometry to find length and angle measures.
12.5.1 By the end of twelfth grade, students will select a sampling technique to gather data, analyze the resulting data and make inferences.
12.5.2 By the end of twelfth grade, students will write equations and make predictions from sets of data.

Contents:
Activity 1– Using Logger Pro for Video Analysis (Page 1)
Activity 2 – Analyze a video of a steel ball hitting a wood “wall”. (Page 9)
Activity 3 – Analyze a video of a vehicle hitting a road barrier. (Page 12)
Follow-up Discussion – What makes an effective road barrier?
Appendices – Solutions and Concluding Comments

Pages numbered 1-15 are designed to be printed for students.

Lesson Organization:
The video and external data for the Activities 1, 2, and 3 was collected by rolling a steel ball down a ramp. A picture of the setup is included in the Appendices. Video of the rolling ball was collected using a digital video camera and converted to Quicktime movie format. A motion detector was used to collect distance vs time data after the ball strikes the wall. Both data sets are analyzed with Logger Pro software available from Vernier (www.vernier.com) which offers a site license for a very nominal fee. The analysis of data involves a discussion of the following topics:

- linear equations,
- quadratic equations,
- parametric equations,
- vectors forming a right triangle,
- parabolic curves,
- percent difference/change,
- piece-wise functions,
- Pythagorean theorem.

Optionally, trigonometric functions can be used to determine angles of impact and reflection.

Activity 3 uses data provided by the Midwest Roadside Safety Facility (MWRSF  http://mwrsf.unl.edu/) at the University of Nebraska-Lincoln. The final activity is more complex because the use of high speed film requires some additional steps when using Logger Pro software for data analysis.
Using Logger Pro for Video Analysis

The final activity involves a discussion of the results of the video analysis. Why are roadside barriers present? What purposes do the barriers serve? What makes an effective roadside barrier? How does the video demonstrate an effective road barrier?

Data Sets
All data is provided in a downloadable file at http://jwelker.lps.org/lessons/videoanalysis.htm
Demonstration
Roll a ball towards a wall or other object so that it hits the wall at an angle other than 90°. Ask students to brainstorm what is happening to the ball. Make as many conjectures as they can. Write them in a list.

Discussion
Imagine the ball was a car and the wall was a bridge railing on the interstate. What would be the important results of a car hitting the bridge railing? (see photo bridge_rail.jpg).

Alternatively, what would you want as the end result of a crash with a roadside barrier such as the ones shown below?

Photo Source:
http://mwrslf.unl.edu/newsletter/MwRSFNewsletter-Issue1.pdf
http://tti.tamu.edu/inside/cdv/cpg/crash.stm
http://tti.tamu.edu/inside/cdv/cpg/images/rombig.jpg
Video Analysis Tutorial – Activity 1

1. Launch Logger Pro

2. Select Insert – Movie. Browse to find the movie Sample01.mov and select it. The movie should be inserted into the Logger Pro window.

3. The movie can be enlarged by clicking on the movie window and dragging one of the corner handles.

The image below describes the tools for analyzing a movie. If the toolbar on the right side is not showing, click the Hide/Show Tools button.

4. If using Windows versions of Logger Pro with a digital video camera or digital movie, right click on the movie and choose the Movie Options. Check the boxes for Correct Aspect Ratio for DV Movies and Deinterlace Movie.

5. Set the scale for the image. The block of wood at the top was measured to be 0.498 m. Select the Set Scale tool. Click on one end of the block and drag the mouse across to the other end of the wood block. After releasing the mouse, enter the values into the pop-up window.

A green line will appear where the scale has been set. If the line does not show, click the “Hide/Show Scale” button.
Using Logger Pro for Video Analysis

6. Set the Origin. The origin can be moved or set at any time by clicking the Set Origin button. The origin will be moved to the location the cursor is clicked. To change the frame of reference or x-axis orientation, drag the circle located on the x-axis to a new location. Data points previously entered are automatically adjusted to the new origin and orientation.

   The yellow lines indicate the x- and y-axes.

7. Mark the data points. Click the Set/Add Points button and carefully click on the steel ball in the lower right corner of the window. After clicking, the movie will move to the next frame and you may repeat the process. Be careful not to click on the window with the Add/Set Point button selected as erroneous points will be entered.

   The first set of data points is typically marked with blue dots on the movie window. As data points are added, the values are stored in the table and graphed in the graph window. The x-values on the graph are red and the y-values are blue. Continue clicking on the object until all of the desired values have been entered.

8. Resize the graph, data, and movie window by clicking on the window and dragging the size boxes of the window. To move a window, position the cursor over the edge of the window until the cursor becomes a “hand” cursor. Click and drag the window to a new location. The resulting movie and graph windows may be similar to those shown below.

   Note: The origin as been moved using the set origin (Instruction 6 above) in the photo below.
9. Synchronize the video to the data. Click the **Rewind** button to move the selected point to the start of the data. Click the sync button. Both values will show 0 if you started the first point on the first frame of the video.

**Windows Version:** Enter the graph time which matches the point selected on the movie.

**Macintosh Version:** You must enter the time for the movie point selected.

10. Examine the video and graph together. Click the Examine button in the menu.

Position the cursor over the graph. Move the cursor and watch the movie points appear synchronized with the cursor passing over the points on the graph. The values will appear in a text box which may be moved by positioning the cursor over the border until the “hand” cursor appears. Click and drag the box to a new position or close the box as necessary.
Analyzing the Data

The data collected has two distinct parts. Both may be modeled using Logger Pro. Mathematically, the models will be reported as a piece-wise parametric function. The two distinct parts of the graph consist of the portion before the ball hits the wall and the portion after the ball hits the wall.

1. Click and drag the cursor on the graph to highlight the region to analyze. The highlighted or shaded region appears in the image at the left.
2. Choose Linear Regression from the toolbar or Analyze-Linear Fit from the file menu.

Select both the “X” and “Y” values in the check box menu.
3. The regression boxes may be moved by position the cursor over the edge until the “hand” cursor appears. Click and drag the box to the desired location.

Repeat steps 1 and 2 to select the portion of the graph after the ball strikes the wall.

Models:
\[ X(t) = -0.314_{(m/s)} t + 0.156_{(m)} \]
\[ Y(t) = 0.415_{(m/s)} t + 0.033_{(m)} \]
Where \( t \leq 0.502_{(s)} \)

\[ X(t) = -0.211_{(m/s)} t + 0.091_{(m)} \]
\[ Y(t) = -0.118_{(m/s)} t + 0.291_{(m)} \]
Where \( t > 0.502_{(s)} \)

**Question A:**
What do the slopes “m” and “y”-intercepts or “b” of each model indicate?

**Question B:**
What is the speed of the ball after striking the wall?

**Question C:**
Predict the position of the ball at 0.230 seconds and at 2.395 seconds.
Starting a New Page

If your screen is full of graphs, movies, and data, Logger Pro offers the opportunity to add a “new page” to the document. From the “Page” menu, choose “Add Page”. From the window, choose “Copy Current Page” and click OK.

We will continue using the movie and the data set on this new page. To make room for new graphs, delete the graph on this new page by clicking on it and pressing the delete key.

Move back and forth between the pages by using the Page – Previous Page – Next Page and Go To Page menus.

Importing and Synchronizing External Data

As the video was being collected, a motion detector was set to measure the distance from the steel ball to the motion detector after the ball struck the wall. (See the Appendices for an image of the setup.) The motion detector data in the file sample01.csv can be imported and synchronized with the video.

1. From the File – Import From menu, choose “Text File”. Browse to find the file “sample01.csv” and select it.
2. When the data is imported, the column headings are not present. Slide the data window to view imported data. Double click to edit the column headings. The first column is “Time” with seconds as the units. The second column is “Distance” in meters.

A graph will also appear with the data. Once the column headings are updated, the graph labels will automatically update.

The graph also may not show the origin. Right-click (Mac users ctrl-click) on the graph and choose AutoScale – AutoScale from 0 when the pop-up menu opens.

The graph of the data will be similar to the one on the following page.
The imported motion detector data graph should be similar to the graph at the left.

Fit a linear model to the data imported from the motion detector just as you did for the VideoAnalysis data.

**Question D:**
What do the “m” and “b” values mean for this data.

**Question E:**
What is the percent difference of answer to Question B and this answer for the speed of the ball after striking the wall?

**Windows Users Only (Macintosh version will not synchronize to an external data set.)**

3.. Synchronizing the data requires that we know the time of the imported data and the time of the video when the steel ball struck the wall. In this case, the first value of the imported data is 0.32 seconds and was provided to us as the time the ball struck the wall.

Use the frame slider and previous/next buttons on the movie window to position the ball at the point of impact. Record the time from the movie window at the point where the ball strikes the wall. The move time is in the upper right corner of the movie window.

With the point on the video still selected, click the Synchronize button on the video window. Change the data set to synchronize to “Sample01”. Check the Synchronize to this data set box. Enter the time of the graph recorded previously into the “Graph Time” box. Windows users may click in the “Graph Time” box and then click the matching point on the graph (for this graph, it is the first point).

The movie should now be synchronized with the “Sample01.csv” data collected from the motion detector.

To return the synchronization to the X-Y video data set on Page 1 of the Logger Pro file, repeat the process with the movie set to the first point, select VideoAnalysis in the dropdown box and enter the graph time as “0”. (Windows users may simply enter “0” and “0” in both the Graph Time and Movie Time boxes because the video was synchronized with the starting time at zero.)
Using Logger Pro for Video Analysis – Activity 2

Start a new Logger Pro document. Analyze the video and external data using the methods discussed in the previous tutorial. The files for this activity are:
- Movie – Sample02.mov
- External Data – Sample02.csv

1. Use video analysis and write a piece-wise parametric model.
   a. Provide a model with units for the steel ball prior to hitting the wood “wall”.
      \[ X_1(t) = \text{____________________________} \]
      \[ Y_1(t) = \text{____________________________} \]
      \[ t \text{ is valid } \text{____________________________} \]
   b. Provide a model with units for the steel ball after hitting the wood “wall”.
      \[ X_2(t) = \text{____________________________} \]
      \[ Y_2(t) = \text{____________________________} \]
      \[ t \text{ is valid } \text{____________________________} \]

      Combine the velocity in the x- and y- directions to determine the speed of the ball after impacting the wall. __________

   c. Analyze the external data and determine a distance versus time model for the ball after striking the wall.
      Speed from the model __________

      \% difference of speed in part ( b ) compared to speed in part ( c ). __________

2. Compare the velocities calculated from the external data of Sample01 and Sample02. How are they similar? How are they different?

3. Estimate the velocity of the ball prior to impact with the “wall”.

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4. Optionally: Use trigonometric functions to determine the angle of impact ($\theta_1$) with the wall and the angle of reflection ($\theta_2$) with the wall. Use the data from Sample02 to find the angles.

\[ \theta_1 = \text{____________________} \]
\[ \theta_2 = \text{____________________} \]

Find the angles for the tutorial activity 1 - Sample01.

\[ \theta_1 = \text{____________________} \]
\[ \theta_2 = \text{____________________} \]

Is there a pattern?

5a. Print a copy of your own Logger Pro movie image or use the one below. Draw a line of best fit through the points of both parts of the model to the line at the edge of the board. Use a protractor to measure the angle of impact ($\theta_1$) with the wall and the angle of reflection ($\theta_2$) with the wall. Compare (% difference) the calculated angle with the measured angle?

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5b. What would you have predicted about the angle of impact and the angle of reflection? What might cause the angles to be different? (Roll a ball into a wall or object and observe closely what happens.)

6. Imagine the ball as an automobile on a crowded roadway and the wall is a retaining wall running between a multi-lane freeway in a major city.
   a. What would be the result of an automobile bouncing off of the wall?
   b. What could be done to make the result of such a crash safer?
   c. Are there limits to what can be done? Why not remove the wall all together?
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Analyzing Crash Test Video – Activity 3

Analyze the crash test video from the Midwest Road Safety Institute at the University of Nebraska - Lincoln. The video demonstrates a pickup truck crashing into a “W” beam road barrier attached to wooden support posts similar to the pictures on page 1 of this document.

1. Insert the video npg4_locam_overhead.mov into a new logger pro file.

2. Position the video with the pickup truck directly under the camera. Set the scale length using the vehicle bumper to bumper length at 18.167 ft.

3. The video was shot at 505 frames per second. Logger Pro assumes the video was 29.97 frames per second. To convert the time, insert a new calculated column. Since the time frame is in 29.97 frames per second, we need a conversion factor “r” to convert to 505 frames per second. \[ 29.97 \times r = 505 \] or \[ r = \frac{505}{29.97} \].

Insert a new calculated column from the Data menu as shown in the image at the right. Name the column “Time Converted” with short name Tc and units of “s” for seconds in the dialog box which appears. Click in the “Equation” box to enter the formula for the conversion. From the “Variables” drop down menu, choose “Time”. The column variable “Time” will be entered into the Equation box. Complete the rest of the equation as “Time”*29.97/505 and click the done button.

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4. Change the x-axis of the graph to indicate “Time Converted” should be placed on the x-axis. Click on the “Time” section of the label. From the menu, choose “Time Converted”.

5. Use the “Add Point” button to mark the position of the crash test marker on the hood of the truck. Enlarge the video as much as possible. Due to the high speed video, you may choose to click the frame advance button 3 or 4 times between each point. This will make locating the position of the marker easier and more consistent for each point. Click on a consistent black marker section. Also, some frames may not have a clearly visible mark due to the sun and brightness of the movie.

6. Continue marking data points until the marker on the hood is no longer visible.
NPG4 Worksheet – Activity 3

After plotting the data for the NPG4 crash test, answer the following questions.

1. Compare and contrast the parametric graph of the crash test with the ball bouncing off of the wall in the previous activity. How are the graphs similar? How are they different?

2. The beginning and ending portions of the data appear to be linear. Find a mathematical model for the linear portions of the graph.

Prior to Impact:
\[ X_1(t) = \text{________________________} \]
\[ Y_1(t) = \text{________________________} \]
\[ t \text{ is valid } \text{________________________} \]
Use the x- and y-velocity to estimate the speed of the vehicle \[ \text{______________} \]
Convert the speed of the vehicle to miles per hour. \[ \text{______________} \]

After Impact
\[ X_2(t) = \text{________________________} \]
\[ Y_2(t) = \text{________________________} \]
\[ t \text{ is valid } \text{________________________} \]
Use the x- and y-velocity to estimate the speed of the vehicle \[ \text{______________} \]
Convert the speed of the vehicle to miles per hour. \[ \text{______________} \]
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3. If you were designing a barrier to prevent a vehicle from running off of the road, what would be the goals of the barrier?

4. In which direction (x or y) does the velocity change the most? Is this a positive or negative result? Explain.

5. How does the data from this test demonstrate an effective road barrier?

6. The middle portion of the data set appears to be what shape of curve (particularly the y-values)? Would you expect this type of curve? Why or why not?

7. Watch the road barrier while playing the video.
   a. What happens to the barrier system? Did the barrier function properly? Explain.

   b. Should the barrier be more rigid? What would happen if the barrier was more rigid? What would happen if it were less rigid? Explain
Appendices

Setup Photo

Activity 1 Solutions

Activity 2 Solutions

Activity 3 Solutions
A steel ball was allowed to roll down the “ramp”. The video camera recorded its motion as it rolled toward the “wall” and bounced off the “wall” toward the motion detector. The distance from the motion detector was recorded as the ball proceeded towards the detector. The data points before striking the wall and after the ball was too close to the detector were manually removed from the data set.

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Solutions to Tutorial Questions – Activity 1

**Question A:**

The data is provided in parametric form meaning there is an equation for x position in terms of time “X(t)” and an equation for the y position in terms of time “Y(t)”.

\[ X(t) = -0.314_{(m/s)} t + 0.156_{(m)} \]

Slope is \( \frac{\Delta y}{\Delta x} \) where \( y \) is the vertical axis and \( x \) is the horizontal axis. This equation has “X” on the vertical axis and “Time” on the x-axis. Therefore, the slope of this graph is the change in the “x” or horizontal direction of the video divided by the change in time. The units will be meters divided by seconds (m/s) or velocity in the “X” direction. Since the velocity is negative, the ball was moving at 0.314 m/s to the left.

The “b” value is the position of the object on the vertical axis when the value on the horizontal axis is zero. In this case, the “X” position of the ball when the time is 0 would be predicted to be 0.156 m.

\[ Y(t) = 0.415_{(m/s)} t + 0.033_{(m)} \]

Similarly, the velocity in the “y” direction will be 0.415 m/s. Since the slope is positive, the ball is moving upward on the video which is the positive “y” direction. The “b” value of 0.033 m indicates the ball was 0.033 m in the “y” direction when \( t \) was 0.

The second parametric equation is evaluated in a similar manner. However, the interpretation of the “b” value is a bit different.

\[ X(t) = -0.211_{(m/s)} t + 0.091_{(m)} \]

The slope of -0.211(m/s) indicates the ball is moving to the horizontally in the “x” direction to the left at a speed of 0.211(m/s). **If the ball had been in continuous motion** meaning it didn’t bounce off of the board, it would have to been at an “x” position of 0.091(m) when the time was zero.

\[ Y(t) = -0.118_{(m/s)} t + 0.291_{(m)} \]

The slope of -0.118(m/s) indicates the ball is moving to the vertically down in the “y” direction at a speed of 0.118(m/s). **If the ball had been in continuous motion** meaning it didn’t bounce off of the board, it would have to been at a “y” position of 0.291(m) when the time was zero.

When \( t=0 \), the second parametric model predicts the location of the object would have been at (0.091, 0.291).
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**Question B:**

The equations provide the velocity in the “x” and “y” directions. Using vectors and the Pythagorean theorem, it is possible to calculate the velocity of the ball.

![Diagram of vectors](image)

The sum of perpendicular vectors $\vec{x}$ and $\vec{y}$ is the resultant vector $\vec{r}$. The magnitude of $\vec{r}$ is:

$$|\vec{r}| = \sqrt{|x|^2 + |y|^2}$$

$$|\vec{r}| = \sqrt{(-0.211\text{ (m/s)})^2 + (-0.118\text{ (m/s)})^2}$$

$$|\vec{r}| = 0.242\text{ (m/s)}$$

The speed of the ball after hitting the wall is estimated to be 0.242 m/s.

**Question C:**

Since $t = 0.230$ seconds, the time is valid in the first parametric model.

$X(0.230) = -0.314\text{ (m/s)} (0.230\text{ (s)}) + 0.156\text{ (m)} = 0.084\text{ (m)}$

$Y(0.230) = 0.415\text{ (m/s)} (0.230\text{ (s)}) + 0.033\text{ (m)} = 0.128\text{ (m)}$

The model predicts the ball will be at (0.084 m, 0.128 m) when $t = 0.230$ seconds.

When $t = 2.395$ seconds, the second parametric model must be used.

$X(2.395) = -0.211\text{ (m/s)} (2.395\text{ (s)}) + 0.091\text{ (m)} = -0.414\text{ (m)}$

$Y(2.395) = -0.118\text{ (m/s)} (2.395\text{ (s)}) + 0.291\text{ (m)} = 0.008\text{ (m)}$

The model predicts the ball will be at (-0.414 m, 0.008 m) when $t = 2.395$ seconds.

The two positions have been plotted on the image from Logger Pro.

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Question D:

Recall this graph is the distance of the ball measured from the motion detector with distance on the y-axis and time on the x-axis. The slope is $\frac{\Delta y}{\Delta x}$ or $\frac{\Delta d}{\Delta t}$ which is velocity. The velocity of the ball is 0.2343 m/s moving towards the detector since the distance is decreasing (negative slope). The y-intercept or “b” value would be the distance from the motion detector when the time is zero. Therefore, the model predicts the distance would have been 0.808 m from the detector when $t = 0$ seconds.

Question E:

The percent difference is given by the formula:

$$\frac{(Experimental - Expected)}{Expected} \times 100 = \%\ difference$$

Using the values from the previous answers, the percent difference is:

$$\frac{(0.242 - 0.234)}{0.234} \times 100 = 3.4\%$$

The percent difference between the two methods of determining the speed of the ball is 3.4%.
Activity 2 Solutions

Note: Student calculations may vary depending upon the placement of the axis on the graph which will have the greatest impact upon the b (Y-Intercept) value of the model. The slopes (velocities) should be similar values.

1. Use video analysis and write a piece-wise parametric model.
   a. Provide a model with units for the steel ball prior to hitting the wood “wall”.
   \[ X_1(t) = -0.576 \text{ (m/s)} \cdot t + 0.562 \text{ (m)} \]
   \[ Y_1(t) = 0.708 \text{ t} + 0.042 \]
   t is valid: \( t \leq 0.42 \text{ seconds} \)

   b. Provide a model with units for the steel ball after hitting the wood “wall”.
   \[ X_2(t) = -0.468 \text{ (m/s)} \cdot t + 0.518 \text{ (m)} \]
   \[ Y_2(t) = -0.228 \text{ (m/s)} \cdot t + 0.388 \text{ (m)} \]
   t is valid: \( t > 0.42 \text{ seconds} \)

   Combine the velocity in the x- and y- directions to determine the speed of the ball after impacting the wall. Use the Pythagorean theorem to find the magnitude of the sum of two vectors.
   \[
   \sqrt{(-0.468)^2 + (-0.228)^2} = 0.521
   \]
   The velocity of the ball is 0.521 m/s after impact with the wall.
c. Analyze the external data and determine a distance versus time model for the ball after striking the wall.

![Graph showing distance versus time model](image)

Velocity from the model: -0.519 m/s
Speed is 0.519 m/s

% difference of speed in part (b) compared to speed in part (c).

\[
\frac{(0.519 - 0.521)}{0.521} \times 100 = -0.4\%
\]

2. Compare the velocities calculated from the external data of Sample01 and Sample02. Tutorial. How are they similar? How are they different?

The ball is traveling in the same direction as both velocities are negative. The speed of the ball in Sample01 was 0.23 m/s with a speed of 0.52 m/s in Sample02. Hence the ball was traveling more than twice (2.2 times) as fast in the second sample.

3. Estimate the velocity of the ball prior to impact with the “wall”.

The \( \vec{x} \) and \( \vec{y} \) components add together to form \( \vec{r} \). Using the Pythagorean Theorem:

\[
\sqrt{(-0.576)^2 + (0.708)^2} = 0.913
\]

The velocity is predicted to be 0.913 m/s prior to impacting the wall.
4. Optionally: Use trigonometric functions to determine the angle of impact ($\theta_1$) with the wall and the angle of reflection ($\theta_2$) with the wall.

Using right triangle trig, we know that

$$\tan \theta = \frac{\Delta y}{\Delta x}.$$

The inverse trig function is used to solve for the angle. Therefore, the angle is

$$\theta = \tan^{-1} \left( \frac{\Delta y}{\Delta x} \right).$$

We will find the angle (reference angle) and not find the angle in standard position. As a result, we can use the absolute value of the velocity or speed in determining the angle of impact.

$$\theta_1 = \tan^{-1} \left( \frac{\Delta y}{\Delta x} \right) = \tan^{-1} \left( \frac{0.708}{0.576} \right) = 50.9^\circ$$

$$\theta_2 = \tan^{-1} \left( \frac{\Delta y}{\Delta x} \right) = \tan^{-1} \left( \frac{0.228}{0.468} \right) = 26.0^\circ$$

Find the angles for Activity 1 Sample01.

$$\theta_1 = \tan^{-1} \left( \frac{0.415}{0.314} \right) = 52.9^\circ$$

$$\theta_2 = \tan^{-1} \left( \frac{0.118}{0.211} \right) = 29.2^\circ$$

Is there a pattern?

The angle of impact for both examples is always greater than the angle of reflection. The ratios of $\frac{\theta_1}{\theta_2}$ are 1.96 and 1.85.
5. Print a copy of your own Logger Pro movie image or use the one below. Draw a line of best fit through the points of both parts of the model to the line at the edge of the board. Use a protractor to measure the angle of impact ($\theta_1$) with the wall and the angle of reflection ($\theta_2$) with the wall. Compare (% difference) the calculated angle with the measure angle?

\[ \frac{(52.9 - 50)}{50} \times 100 = 5.8\% \text{ The difference in } \theta_1 \text{ is } 5.8\%. \]
\[ \frac{(29.2 - 25)}{25} \times 100 = 16.8\% \text{ The difference in } \theta_2 \text{ is } 16.8\% \text{ which seems quite high. When comparing differences, the same difference value divided by a smaller number will have a higher } \% \text{ difference. The difference in the angles from the measured values are } 2.9^\circ \text{ and } 4.2^\circ. \]
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5b. What would you have predicted about the angle of impact and the angle of reflection? What might cause the angles to be different? (Roll a ball into a wall or object and observe closely what happens.)

Most students predict the ball will bounce off of the wall with an angle of reflection equal to the angle of impact. Roll a volleyball or basketball against a wall. Close watch the spin of the ball. It soon becomes clear that the spinning of the ball changes upon impact with the wall and is a factor in changing the angle of reflection.

6. Imagine the ball as an automobile on a crowded roadway and the wall is a retaining wall running between a multi-lane freeway in a major city.
   a. What would be the result of an automobile bouncing off of the wall?

      An automobile bouncing off a retaining wall and proceeding back into the lanes of traffic is likely to cause further crashes with the possibility of further injury to the occupant of the vehicle or occupants of other vehicles which are involved in the crash. There is also the great potential for further costly damage to vehicles.

   b. What could be done to make the result of such a crash safer?

      Design the barrier in such a manner that the vehicle stays near the barrier and does not bounce back into the other lanes of traffic. It must also stop the vehicle in a safe manner. Slowly decelerating the vehicle would be preferable to a sudden stop.

   c. Are there limits to what can be done? Why not remove the wall all together?

      The barrier is necessary to prevent head-on collisions with other oncoming vehicles or to prevent the vehicle from exiting the roadway into culverts or ravines which could result in greater injury or damage.
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NPG4 Worksheet – Activity 3 Solutions

Note: There may be wide variation in some answers due to the placement of the origin and the scale.

After plotting the data for the NPG4 crash test, answer the following questions.

1. Compare and contrast the parametric graph of the crash test with the ball bouncing off of the wall in the previous activity. How are the graphs similar? How are they different?

First, notice the paths. The ball bounced directly off of the wall. The path before impact was a straight line, the path after impact was a straight line. The crash test video starts as a linear plot, but the graph is curved after the impact with the guard rail.

This can also be seen in the parametric graph of x (red) and y (blue) positions versus time shown below.

When looking at the graph, a second factor becomes readily apparent. In the ball data the y-axis data made a sudden and sharp change in direction. The position in the x-direction continues moving forward even though the slope of the line is decreasing over time.

The y-axis data shows a change in direction. The truck started vertically about 28 ft from the origin. It was nearly 16 ft from the origin in the y-direction at about 0.5 seconds and then changed direction moving away from the origin. However, the change was much more gradual than shown with the ball data.
Using Logger Pro for Video Analysis

2. The beginning and ending portions of the data appear to be linear. Find a mathematical model for the linear portions of the graph.

Prior to Impact:
\[ X_1(t) = 66.45 \text{ (ft/s)} t - 0.89 \text{ (ft)} . \]
\[ Y_1(t) = -32.63 \text{ (ft/s)} t + 29.41 \text{ (ft)} . \]
\[ t \text{ is valid: } 0 \text{ (s)} \leq t \leq 0.24 \text{ (s)} \]

Use the x- and y-velocity to estimate the speed of the vehicle
\[ v = \sqrt{(66.45)^2 + (-32.63)^2} \]
\[ v = 74.03 \text{ ft/s} \]

Convert the speed of the vehicle to miles per hour.
\[ \left( \frac{74.03 \text{ ft}}{\text{sec}} \right) \left( \frac{1 \text{ mile}}{5280 \text{ ft}} \right) \left( \frac{60 \text{ sec}}{1 \text{ min}} \right) \left( \frac{60 \text{ min}}{1 \text{ hr}} \right) = \]
\[ v = 50.47 \text{ mph} \]
After Impact

\[ X_2(t) = 26.44 \text{ (ft/s)} \times t + 22.70 \text{ (ft) } \]

\[ Y_2(t) = 11.04 \text{ (ft/s)} \times t + 9.31 \text{ (ft) } \]

\[ t \text{ is valid: } 0.74 \text{ (s)} \leq t \leq 0.94 \text{ (s)} \]

Use the x- and y-velocity to estimate the speed of the vehicle.

\[ v = \sqrt{(26.44)^2 + (11.04)^2} \]

\[ v = 28.65 \text{ ft/s} \]

Convert the speed of the vehicle to miles per hour.

\[ \left( \frac{28.65 \text{ ft}}{\text{sec}} \right) \left( \frac{1 \text{ mile}}{5280 \text{ ft}} \right) \left( \frac{60 \text{ sec}}{1 \text{ min}} \right) \left( \frac{60 \text{ min}}{1 \text{ hr}} \right) = \]

\[ v = 19.5 \text{ mph} \]
3. If you were designing a barrier to prevent a vehicle from running off of the road, what would be the goals of the barrier?

- Minimize injury to the passengers of the vehicle.
- Prevent the vehicle from exiting the roadway.
- Prevent the vehicle from returning into the path of another vehicle.
- Minimize cost and time to repair the barrier to prevent another accident after the barrier has been damaged.

4. In which direction (x or y) does the velocity change the most? Is this a positive or negative result? Explain.

The velocity in the x-direction decreased from 66.45 \( (ft/s) \) to 26.44 \( (ft/s) \) which is a change of \((66.45 – 26.44)/66.45*100 = 60.2\%\).

The velocity in the y-direction changed from -32.63 \( (ft/s) \) to 11.04 \( (ft/s) \) or \((-32.63 – 11.04)/(-32.63)*100 = 133.8\%\).

The most dramatic change in velocity occurred in the y-direction. The vehicle continued moving in the x-direction at a decreased speed. However, the vehicle changed from a negative velocity to a positive velocity in the y-direction. The change from positive to negative indicates the change in direction.

5. How does the data from this test demonstrate an effective road barrier?

To prevent injury to the passengers, the goal is to slow the vehicle in a safe manner while controlling its direction. The velocity of the vehicle in the x-direction of this video would be the forward motion of the vehicle continuing down the roadway. Continuing motion in this direction would not endanger the passengers of the vehicle as much as the velocity in the y-direction which is heading the vehicle off of the roadway. The goal is to gradually change the vehicles direction and slow the vehicle at the same time while preventing it from becoming involved with a crash after exiting the barrier. The combination of the barrier redirecting the vehicle while preventing the vehicle from exiting the roadway is effective. The manner in which the vehicle and the roadway barrier system absorb the impact of the crash serve to reduce the potential for injury to the occupant. This is accomplished by slowing the speed of the vehicle and slowly changing the direction of the vehicle. Sudden changes to either the speed of the vehicle or the direction of the vehicle produce greater changes to the velocity. The change in velocity is acceleration which results in a force being applied to the occupants of the vehicle. Minimizing the forces applied to the occupant will likely lead to a decreased risk of injury to the occupant.
6. The middle portion of the data set appears to be what shape of curve (particularly the y-values)? Would you expect this type of curve? Why or why not?

Yes! A linear “curve” of a displacement vs. time graph indicates constant velocity. Since the velocity is changing, the slope of the line cannot be constant. The slope of the tangent line to a curve represents the acceleration at the particular point.

Also, note that the “curved” portions of the x and y data appear to be at different times. The curved portion of the y (square boxed points) data appears to be from about 0.3 seconds to about 0.6 seconds. The data from the x (circled points) appears to start curving between 0.5 ad 0.6 seconds. The vehicles x- and y- velocities were being redirected at different times.

Extending the learning to Quadratic Functions

You may choose to extend the learning at this point for students who have an understanding of quadratic functions and the graph of a parabola. Logger Pro will also fit curves other than linear functions. Select the portion of the graph to fit with the selection cursor. From the menu, choose Analyze – Curve Fit or select the curve fit icon in the toolbar.

Choose the data set to perform the quadratic regression and press OK. Next, select the quadratic regression from the menu of regression choices. Finally, click the “Try Fit” button. Once the curve is fit, the domain values for the fit can be changed by clicking and dragging the [ or ] bars at the end of the data set. This allows the user to change the values over which the quadratic regression is calculated.
Looking at only the y-axis data, one can see a quadratic regression applied to this section of the data. This would also project the continued path which would have taken the vehicle back into the roadway if this path would have continued.

Recall the coefficient “A” in the \( x^2 \) term graphically indicates the amount of “stretching” or how rapidly the curve is increasing.

A quadratic regression has been fit to the data in the “x” direction. Compare the “A” values between the graphs. The negative value represents a reflection of the curve. The parabola from this reflection is opening downward. Compare the absolute value of each “A” value. The value of 40 ft/s\(^2\) in the x-direction is less than the value of 61.7 ft/s\(^2\) in the y-direction, providing further evidence of a more rapid change in velocity in the y-direction than the x-direction.
7. Watch the road barrier while playing the video.
   a. What happens to the barrier system? Did the barrier function properly? Explain.

   The metal “W” beam is deformed but does not break and allow the truck to go through the barrier. It also does not rupture and penetrate into the truck. The wooden support posts break which provide for a gradual as compared to sudden decrease in the velocity of the vehicle.

   b. Should the barrier be more rigid? What would happen if the barrier was more rigid? What would happen if it were less rigid? Explain

   The barrier was appropriate for this type of vehicle. The motion was controlled. If the barrier were more rigid or the vehicle was lighter, it may cause the vehicle to be projected back into traffic on the roadway. If the barrier were less rigid, it may break and penetrate the vehicle or allow the vehicle to leave the roadway surface. Because of the variations in vehicle lengths, heights, and weights, the barrier systems need to be continually studied.

   To extend the learning further, view the video NPG6. The support posts in NPG6 are 24 inches as compared to 73 inches in NPG4 which makes the barrier system much more rigid. Students could analyze this video as well.

**Concluding Comments**

Calculated Speeds: As demonstrated from the external data when rolling the ball, analyzing video can be an extremely accurate method for analyzing motion. It can also be shown that increasing the distance from the camera to the object, the greater the inaccuracy due to a number of reasons. Error results from inaccurately marking the scale setting and the fact that a triangle is formed from the camera facing straight down but the distance is measured at an angle from the camera. To judge the impact, adjust the scale setting in Logger Pro. A small change in the scale setting has a marked impact on the velocity of the truck. To provide a measure of the magnitude of the error, MWRSF data indicates the truck crash speed was actually 60 mph.

After seeing the high speed video played at slower speeds on the computer, it may be important to share with students the critically important function of crash testing and the dangers inherent with operating a motor vehicle. Suggested concluding activities:

1. Play the NPG4 ground.mov quicktime movie. This movie is played at regular speed and shown from ground level. It is a realistic view of the impact which is not observed from the overhead camera. The impact of the crash is shown in this view.
2. Play the NPG2 fail.mov quicktime movie. The movie shows a failed test as the pickup truck breaks through the roadway barrier and becomes a rollover incident. This movie shows that testing is critically important and the fact that operating a motor vehicle can be dangerous. It is extremely important to continue testing road design safety features as vehicles and roadways continue to change with time.
3. Students with math and science skills interested in these types of activities should contact the UNL College of Engineering.